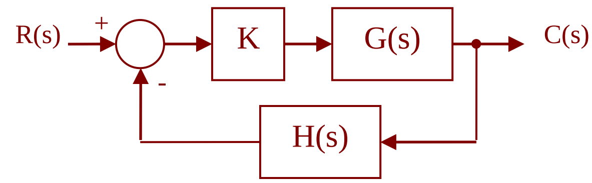
Rules for Making Root Locus Plots



The closed loop transfer function of the system shown is

$$T(s)=\frac{KG(s)}{1+KG(s)H(s)}$$

So the characteristic equation (*c.e.*) is

$$1+KG(s)H(s)=1+K\frac{N(s)}{D(s)}=0\text{, or }D(s)+K\,N(s)=0.$$

As K changes, so do locations of closed loop poles (i.e., zeros of *c.e.*). The table below gives rules for sketching the location of these poles for $K=0\rightarrow\infty$ (i.e., $K\geq0$).

Rule Name	Description
Definitions	• The loop gain is KG(s)H(s) or $K\frac{N(s)}{D(s)}$. • N(s), the numerator, is an m^{th} order polynomial; D(s), is n^{th} order. • N(s) has zeros at z_i (i=1..*m*); D(s) has them at p_i (i=1..*n*). • The difference between *n* and *m* is *q*, so *q=n-m*. ($q\geq0$)
Symmetry	The locus is symmetric about real axis (i.e., complex poles appear as conjugate pairs).
Number of Branches	There are *n* branches of the locus, one for each closed loop pole.
Starting and Ending Points	The locus starts (K=0) at poles of loop gain, and ends ($K\rightarrow\infty$) at zeros. Note: this means that there will be *q* roots that will go to infinity as $K\rightarrow\infty$.
Locus on Real Axis*	The locus exists on real axis to the left of an odd number of poles and zeros.
Asymptotes as $\lvert s\rvert\rightarrow\infty$*	If q>0 there are asymptotes of the root locus that intersect the real axis at $\sigma=\frac{\sum_{i=1}^{n}p_i-\sum_{i=1}^{m}z_i}{q}$, and radiate out with angles $\theta=\pm r\frac{180}{q}$, where *r*=1, 3, 5…
Break-Away/-In Points on Real Axis	Break-away or –in points of the locus exist where N(s)D'(s)-N'(s)D(s)=0.
Angle of Departure from Complex Pole*	Angle of departure from pole, p_j is $\theta_{depart,p_j}=180^\circ+\sum_{i=1}^{m}\angle(p_j-z_i)-\sum_{i=1,\,i\neq j}^{n}\angle(p_j-p_i)$.
Angle of Arrival at Complex Zero*	Angle of arrival at zero, z_j, is $\theta_{arrive,z_j}=180^\circ-\sum_{i=1,\,i\neq j}^{m}\angle(z_j-z_i)+\sum_{i=1}^{n}\angle(z_j-p_i)$.
Locus Crosses Imaginary Axis	Use Routh-Hurwitz to determine where the locus crosses the imaginary axis.
Given Gain "K," Find Poles	Rewrite *c.e.* as D(s)+KN(s)=0. Put value of K into equation, and find roots of *c.e.*. (This may require a computer)
Given Pole, Find "K."	Rewrite *c.e.* as $K=-\frac{D(s)}{N(s)}$, replace "s" by desired pole location and solve for K. Note: if "s" is not exactly on locus, K may be complex (small imaginary part). Use real part of K.

*These rules change to draw complementary root locus ($K\leq0$). See next page for details.

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Complementary Root Locus

To sketch complementary root locus (K≤0), most of the rules are unchanged except for those in table below.

Rule Name	Description
Locus on Real Axis	The locus exists on real axis to the right of an odd number of poles and zeros.
Asymptotes as $\|s\|\rightarrow\infty$	If q>0 there are asymptotes of the root locus that intersect the real axis at $\sigma = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{q}$, and radiate out with angles $\theta = \pm p \frac{180}{q}$, where p=0, 2, 4…
Angle of Departure from Complex Pole	Angle of departure from pole, p_j is $\theta_{depart,p_j} = \sum_{i=1}^{m} \angle(p_j - z_i) - \sum_{i=1, i\neq j}^{n} \angle(p_j - p_i)$.
Angle of Departure at Complex Zero	Angle of arrival at zero, z_j, is $\theta_{arrive,z_j} = \sum_{i=1, i\neq j}^{m} \angle(z_j - z_i) - \sum_{i=1}^{n} \angle(z_j - p_i)$.

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