Z Transform Pairs and Properties

Z Transform Pairs		
Time Domain *	**Z Domain**	
	z	**z^{-1}**
δ[k] (unit impulse)	1	1
γ[k] † (unit step)	$\Gamma(z)=\frac{z}{z-1}$	$\Gamma(z)=\frac{1}{1-z^{-1}}$
a^k	$\frac{z}{z-a}$	$\frac{1}{1-z^{-1}a}$
e^{-bTk}	$\frac{z}{z-e^{-bT}}$	$\frac{1}{1-z^{-1}e^{-bT}}$
k	$\frac{z}{(z-1)^2}$	$\frac{z^{-1}}{(1-z^{-1})^2}$
sin(bk)	$\frac{z\sin(b)}{z^2-2z\cos(b)+1}$	$\frac{z^{-1}\sin(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
cos(bk)	$\frac{z(z-\cos(b))}{z^2-2z\cos(b)+1}$	$\frac{1-z^{-1}\cos(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
$a^k\sin(bk)$	$\frac{az\sin(b)}{z^2-2az\cos(b)+a^2}$	$\frac{az^{-1}\sin(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$
$a^k\cos(bk)$	$\frac{z(z-a\cos(b))}{z^2-2az\cos(b)+a^2}$	$\frac{1-az^{-1}\cos(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$

*All time domain functions are implicitly=0 for k<0 (i.e. they are multiplied by unit step, γ[k]).

†u[k] is more commonly used for the step, but is also used for other things. γ[k] is chosen to avoid confusion (and because in the Z domain it looks a little like a step function, Γ(z)).

Z Transform Properties	
Property Name	**Illustration**
Linearity	$af_1[k]+bf_2[k] \overset{Z}{\longleftrightarrow} aF_1(z)+bF_2(z)$
Left Shift by 1	$f[k+1] \overset{Z}{\longleftrightarrow} zF(z)-zf[0]$
Left Shift by 2	$f[k+2] \overset{Z}{\longleftrightarrow} z^2F(z)-z^2f[0]-zf[1]$
Left Shift by n	$f[k+n] \overset{Z}{\longleftrightarrow} z^nF(z)-z^n\sum_{k=0}^{n-1}f[k]z^{-k}$ $= z^n\left(F(z)-\sum_{k=0}^{n-1}f[k]z^{-k}\right)$
Right Shift by n	$f[k-n] \overset{Z}{\longleftrightarrow} z^{-n}F(z)$
Multiplication by time	$kf[k] \overset{Z}{\longleftrightarrow} -z\frac{dF(z)}{dz}$
Scale in z	$a^kf[k] \overset{Z}{\longleftrightarrow} F\left(\frac{z}{a}\right)$
Scale in time	$f\left[\frac{k}{n}\right] \overset{Z}{\longleftrightarrow} F(z^n)$; n is an integer $n \geq 1$
Convolution	$f_1[k]*f_2[k] \overset{Z}{\longleftrightarrow} F_1(z)F_2(z)$
Initial Value Theorem	$f[0]=\lim_{z\to\infty}F(z)$
Final Value Theorem (if final value exists)	$\lim_{k\to\infty}f[k]=\lim_{z\to1}(z-1)F(z)$