Fourier Transform Pairs

Some Basic Functions

x(t)	X(ω)
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\omega)e^{j\omega t}d\omega$ (Synthesis)	$X(\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$ (Analysis)
$\delta(t)$ (impulse)	1 (constant)
$\Pi(t)=\begin{cases}0, & \lvert t\rvert>1/2\\ 1, & \lvert t\rvert\le 1/2\end{cases}$ (unit rectangular pulse, width=1)	$\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$ (sinc)
1 (constant)	$2\pi\delta(\omega)$ (impulse)
$e^{j\omega_0 t}$ (complex exponential)	$2\pi\delta(\omega-\omega_0)$ (shifted impulse)
$e^{-\alpha t}\gamma(t)$ (causal exponential)	$\frac{1}{j\omega+\alpha}$ (same as Laplace w/ s=jω)
$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{t^2}{2\sigma^2}}$ (Gaussian)	$e^{-\frac{\sigma^2\omega^2}{2}}$ (Gaussian)

Derived Functions (using basic functions and properties)

x(t)	X(ω)
$\Pi\left(\frac{t}{T_p}\right)=\begin{cases}0, & \lvert t\rvert>T_p/2\\ 1, & \lvert t\rvert\le T_p/2\end{cases}$ (time scaled rectangular pulse, width=T_p)	$T_p\,\mathrm{sinc}\left(\frac{T_p\omega}{2\pi}\right)$
$\frac{\Omega_p}{2\pi}\mathrm{sinc}\left(\frac{\Omega_p t}{2\pi}\right)$	$\Pi\left(\frac{\omega}{\Omega_p}\right)$ (rectangular pulse in ω)
$\Lambda(t)=\begin{cases}0, & t>1\\ \lvert 1-t\rvert, & \lvert t\rvert\le 1\end{cases}$ (triangular pulse, width=2)	$\mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$
$\Lambda(t/T_p)$ (scaled triangular pulse, width=$2T_p$)	$T_p\,\mathrm{sinc}^2\left(\frac{T_p\omega}{2\pi}\right)$
$\cos(\omega_0 t)$	$\pi\left(\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right)$
$\sin(\omega_0 t)$	$j\pi\left(\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right)$
$\gamma(t)$ (unit step function)	$\frac{1}{j\omega}+\pi\delta(\omega)$

Using these functions and some Fourier Transform Properties (next page), we can derive the Fourier Transform of many other functions.

Information at http://lpsa.swarthmore.edu/Fourier/Xforms/FXUseTables.html

Fourier Transform Properties

Name	Time Domain	Frequency Domain
Linearity	$\alpha \cdot x_1(t) + \beta \cdot x_2(t)$	$\alpha \cdot X_1(\omega) + \beta \cdot X_2(\omega)$
Time Scaling	$x(t/a)$	$aX(\omega a)$
Time Delay (or advance)	$x(t-a)$	$X(\omega)e^{-j\omega a}$
Complex Shift	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
Time Reversal	$x(-t)$	$X(-\omega)$
Convolution	$x(t) * h(t)$	$X(\omega)H(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(\omega)$
Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Time multiplication	$t^n x(t)$	$j^n \frac{d}{d\omega^n}X(\omega)$
Parseval's Theorem	$\text{Energy} = \int_{-\infty}^{+\infty} \|x(t)\|^2 dt$	$\text{Energy} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \|X(\omega)\|^2 d\omega$
Duality	$X(t)$	$2\pi x(-\omega)$
	$\frac{1}{2\pi}X(-t)$	$x(\omega)$

Symmetry Properties

x(t)	X(ω)
x(t) is real	$X(\omega) = X^*(-\omega)$ $\mathrm{Re}(X(\omega)) = \mathrm{Re}(X(-\omega))$ $\mathrm{Im}(X(\omega)) = -\mathrm{Im}(X(-\omega))$ $\|X(\omega)\| = \|X(-\omega)\|$ $\angle X(\omega) = -\angle X(-\omega)$ Real part of X(ω) is even, imaginary part is odd
x(t) real, even	$X(\omega) = X(-\omega)$ $\mathrm{Im}(X(\omega)) = 0$ X(ω) is real and even
x(t) real, odd	$\mathrm{Re}(X(\omega)) = 0$ $\mathrm{Im}(X(\omega)) = -\mathrm{Im}(X(-\omega))$ X(ω) is imaginary and odd

Relationship between Transform and Series

If $x_T(t)$ is the periodic extension of x(t) then:

$$c_n = \frac{1}{T}X(n\omega_0)$$

Where c_n are the Fourier Series coefficients of $x_T(t)$ and X(ω) is the Fourier Transform of x(t)

Furthermore

$$X_T(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} c_n \delta(\omega - n\omega_0)$$