Rules for Making Bode Plots

Term	Magnitude	Phase
Constant: K	$20\cdot\log_{10}(\lvert K\rvert)$	K>0: 0° K<0: ±180°
Real Pole: $\dfrac{1}{\dfrac{s}{\omega_0}+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -20 dB/dec • Connect asymptotic lines at ω_0,	• Low freq. asymptote at 0°. • High freq. asymptote at -90°. • Connect with straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$.
Real Zero*: $\dfrac{s}{\omega_0}+1$	• Low freq. asymptote at 0 dB • High freq. asymptote at +20 dB/dec. • Connect asymptotic lines at ω_0.	• Low freq. asymptote at 0°. • High freq. asymptote at +90°. • Connect with line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$.
Pole at Origin: $\dfrac{1}{s}$	• -20 dB/dec; through 0 dB at $\omega=1$.	• Line at -90° for all ω.
Zero at Origin*: s	• +20 dB/dec; through 0 dB at $\omega=1$.	• Line at +90° for all ω.
Underdamped Poles: $\dfrac{1}{\left(\dfrac{s}{\omega_0}\right)^2+2\zeta\left(\dfrac{s}{\omega_0}\right)+1}$	• Low freq. asymptote at 0 dB. • High freq. asymptote at -40 dB/dec. • Connect asymptotic lines at ω_0. • Draw peak† at freq= ω_0, with amplitude $H(j\omega 0)=-20\cdot\log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at -180°. • Connect with line from $\omega=\omega_0\cdot 10^{-\zeta}$ to $\omega_0\cdot 10^{\zeta}$
Underdamped Zeros*: $\left(\dfrac{s}{\omega_0}\right)^2+2\zeta\left(\dfrac{s}{\omega_0}\right)+1$	• Low freq. asymptote at 0 dB. • High freq. asymptote at +40 dB/dec. • Connect asymptotic lines at ω_0. • Draw dip† at freq= ω_0, with amplitude $H(j\omega 0)=+20\cdot\log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at +180°. • Connect with line from $\omega=\omega_0\cdot 10^{-\zeta}$ to $\omega_0\cdot 10^{\zeta}$
Time Delay: e^{-sT}	• No change in magnitude	• Phase drops linearly. Phase = $-\omega T$ radians or $-\omega T\cdot 180/\pi$°. On logarithmic plot phase appears to drop exponentially.

Notes:

$\omega 0$ is assumed to be positive. If $\omega 0$ is negative, magnitude is unchanged, but phase is reversed.

* Rules for drawing zeros create the mirror image (around 0 dB, or 0°) of those for a pole with the same ω_0.

† We assume any peaks for $\zeta>0.5$ are too small to draw, and ignore them. However, for underdamped poles and zeros peaks exists for $0<\zeta<0.707=1/\sqrt{2}$ and peak freq. is not exactly at, ω_0 (peak is at $\omega_{peak}=\omega_0\sqrt{1-2\zeta^2}$).

For n^{th} order pole or zero make asymptotes, peaks and slopes n times higher than shown. For example, a double (i.e., repeated) pole has high frequency asymptote at -40 dB/dec, and phase goes from 0 to –180°). Don't change frequencies, only the plot values and slopes.

Tool for learning how to draw Bode plots by hand at: https://lpsa.swarthmore.edu/Bode/bodeDraw.html

More detail at https://lpsa.swarthmore.edu/Bode/Bode.html

Quick Reference for Making Bode Plots

If starting with a transfer function of the form (some of the coefficients b_i, a_i may be zero).

$$H(s) = C\frac{s^n + \cdots + b_1 s + b_0}{s^m + \cdots + a_1 s + a_0}$$

Factor polynomial into real factors and complex conjugate pairs (p can be positive, negative, or zero; p is zero if a_0 and b_0 are both non-zero).

$$H(s) = C \cdot s^p \frac{(s+\omega_{z1})(s+\omega_{z2})\cdots(s^2+2\zeta_{z1}\omega_{0z1}s+\omega_{0z1}^2)(s^2+2\zeta_{z2}\omega_{0z2}s+\omega_{0z2}^2)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots(s^2+2\zeta_{p1}\omega_{0p1}s+\omega_{0p1}^2)(s^2+2\zeta_{p2}\omega_{0p2}s+\omega_{0p2}^2)\cdots}$$

Put polynomial into standard form for Bode Plots.

$$H(s) = C\frac{\omega_{z1}\omega_{z2}\cdots\omega_{0z1}^2\omega_{0z2}^2\cdots}{\omega_{p1}\omega_{p2}\cdots\omega_{0p1}^2\omega_{0p2}^2\cdots}\cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

$$= K \cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

Take the terms (constant, real poles and zeros, origin poles and zeros, complex poles and zeros) one by one and plot magnitude and phase according to rules on previous page. Add up resulting plots.

More detail at https://lpsa.swarthmore.edu/Bode/Bode.html